A Combinatorial Method for 3D Landmark-based Morphometry
Application to the Study of Coronal Craniosynostosis

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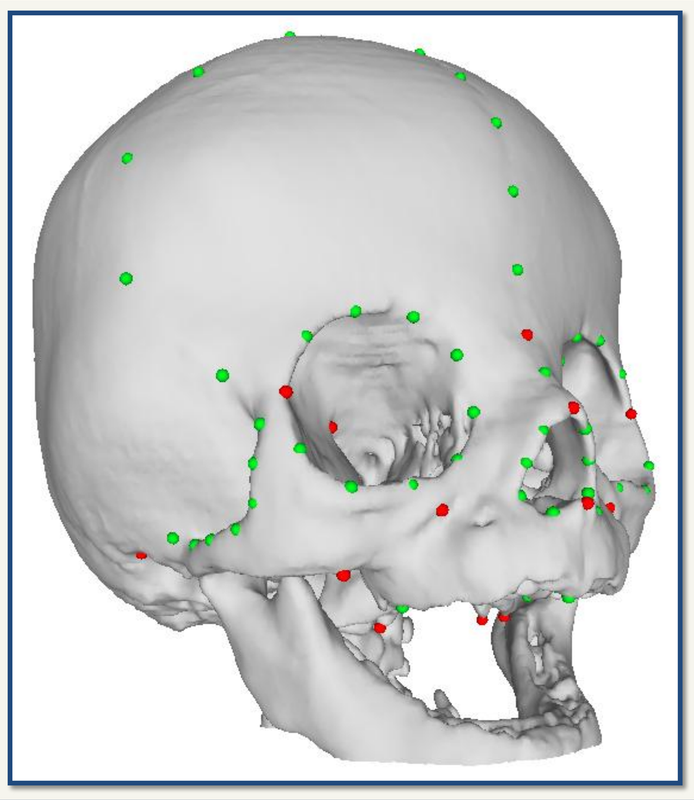
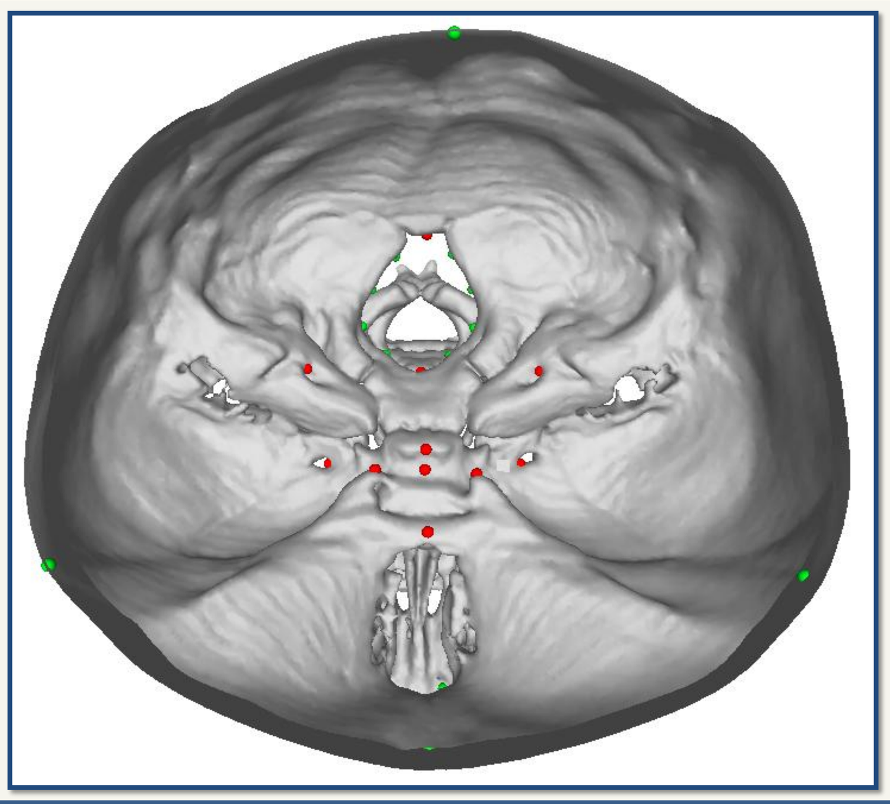
3) CNRS

Thanks to Yann Heuzé and Joan Richtsmeier (Dept of Anthropology, PennState, USA) for access to data and expertise.

- 3D CT images of children (0.1 – 19.9 months):
  - 40 with coronal craniosynostosis (i.e. premature fusion of cranial sutures)
  - 20 unaffected
- Evaluation and classification into 3 diagnosis categories by a clinician:
  - BCS (*bicoronal*): fusion of both lateral sutures (15)
  - LUCS (*left unicoronal*): fusion of only left-side suture (8)
  - RUCS (*right unicoronal*): fusion of only right-side suture (17)
- 133 3D landmarks defined by an expert:
  - 41 anatomical landmarks
  - 92 curve semi-landmarks

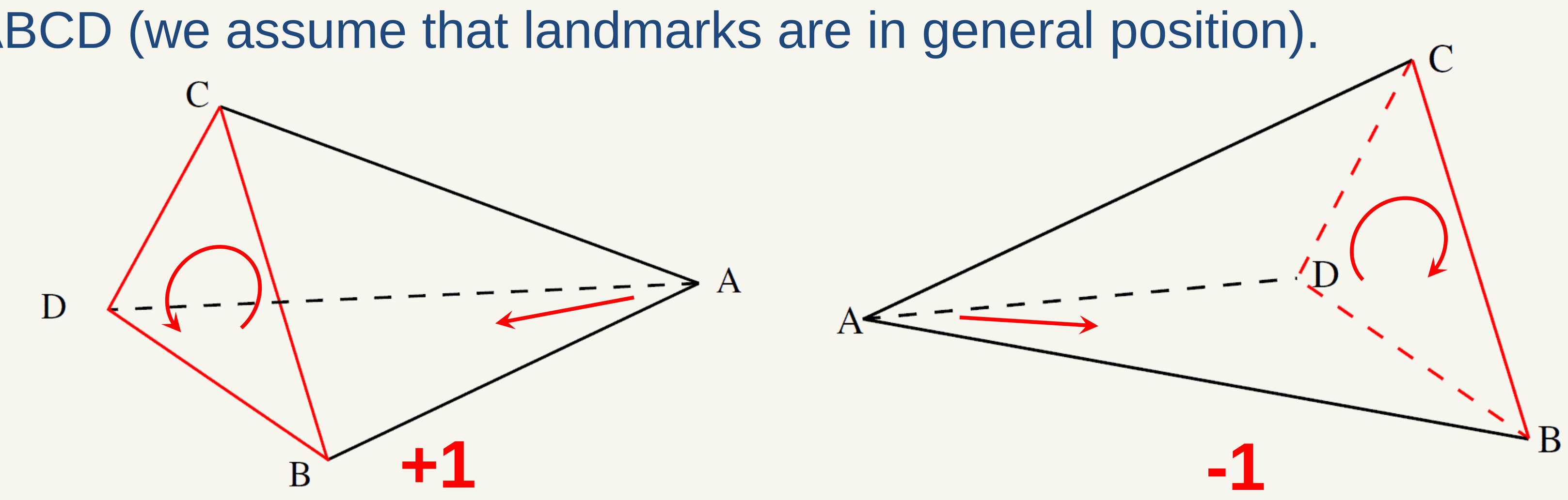




A New 3D Morphometric Method

• Combinatorial Encoding

We call *basis* an ordered 4-uple of 3D landmarks ABCD. Each basis is associated with a sign depending on the orientation of the tetrahedron ABCD (we assume that landmarks are in general position).



For a model $\boldsymbol{M}$ of $\boldsymbol{n}$ landmarks, we can define a set $\mathcal{B}$ of $\binom{n}{4}$ different bases b. The list of their signs forms a vector χ_M, with $\chi_M(b) \in \{-1,1\}$, called **chirotope of *M***, which encodes the **"shape"** of the model.

The properties of χ_M are known as the **oriented matroid theory.** They are only based on the relative positions of landmarks of M and not on any numerical measures as distances or angles.

• Automatic Classification

For a set $\mathcal{M}$ of models $\boldsymbol{M}$ and a subset $\mathcal{C}$ of $\mathcal{M}$, we define the mean $m_{\mathcal{C}}$:

$$m_{\mathcal{C}}(b) = \left(\sum_{M \in \mathcal{C}} \chi_M(b) \right) / |\mathcal{C}|$$

For any subset $\mathcal{S}$ of $\mathcal{B}$, we define a combinatorial distance between **M** and **N**, as a usual distance between their chirotopes:

$$d_{\mathcal{S}}(\chi_M, \chi_N) = \sum_{b \in \mathcal{S}} |\chi_M(b) - \chi_N(b)| \,/\, 2$$

We can classify $\mathcal{M}$ into clusters of models by using this combinatorial distance and, for instance, the K-means criterion/algorithm.

• Characterization of Classes

To characterize a class $\mathcal{C}$, we look for a subset $\mathcal{S}$ of $\mathcal{B}$, the smallest possible, a radius l, and a center x such that $\mathcal{C}$ is contained in

$$B(\mathcal{S}, x, l) = \{ M \in \mathcal{M} \mid d_{\mathcal{S}}(\chi_M, x) \leq l \}$$

and this "ball" separates $\mathcal{C}$ from $\mathcal{M} \backslash \mathcal{C}$.

We sort the bases w.r.t. the value of the discriminability, defined as:

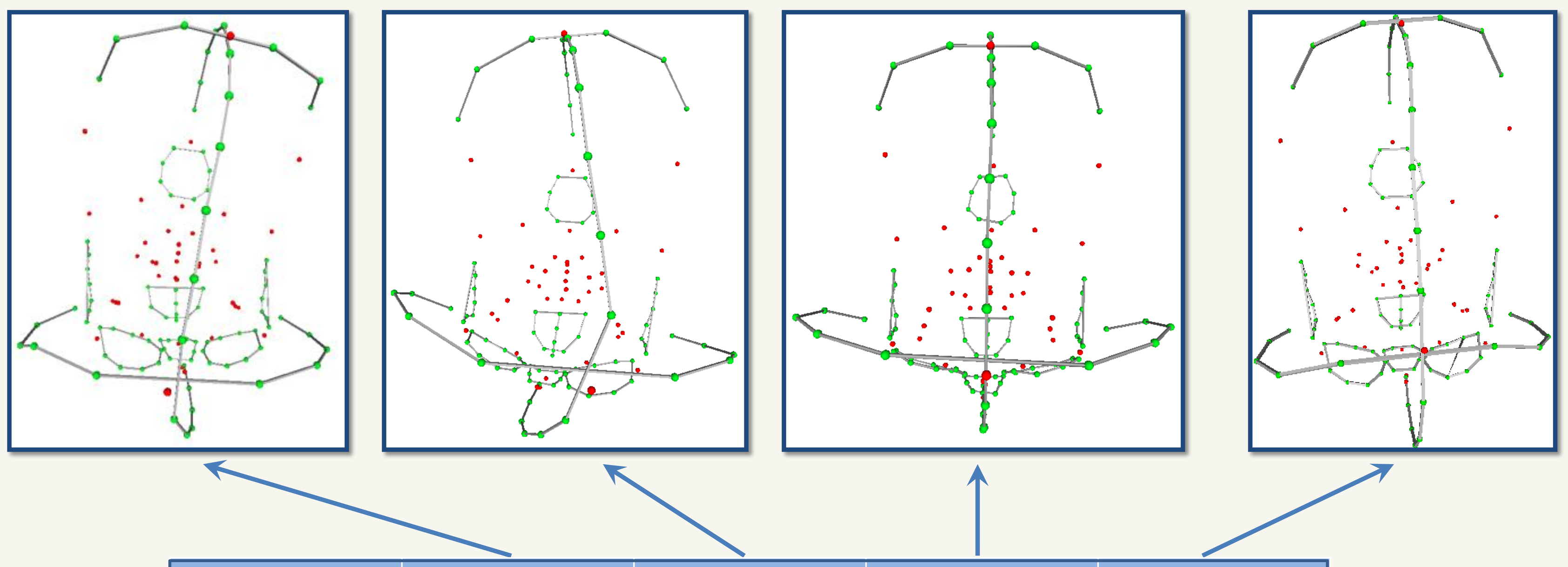
$$\tau(b, \mathcal{C}, \mathcal{M} \backslash \mathcal{C}) = |\, m_{\mathcal{C}}(b) - m_{\mathcal{M} \backslash \mathcal{C}}(b) \,| \,/\, 2$$

The closer to 1 the discriminability is, the more significant the basis b is to characterize the class $\mathcal{C}$. We look for the bases of $\mathcal{S}$ among those of $\mathcal{B}$ with the highest discriminability.

Research supported by the OMSMO project (LIRMM), the TEOMATRO grant ANR-10-BLAN-0207, and the French-South African INLOO project (CNRS).

Application to Craniosynostosis

• Automatic Classification

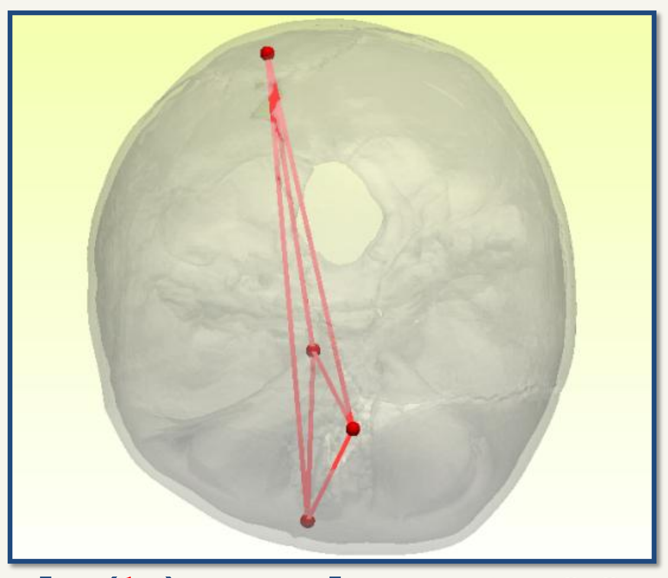


	LUCS	RUCS	BCS	Unaffected
Cluster 1	8	0	0	0
Cluster 2	0	17	0	0
Cluster 3	0	0	15	0
Cluster 4	0	0	0	20

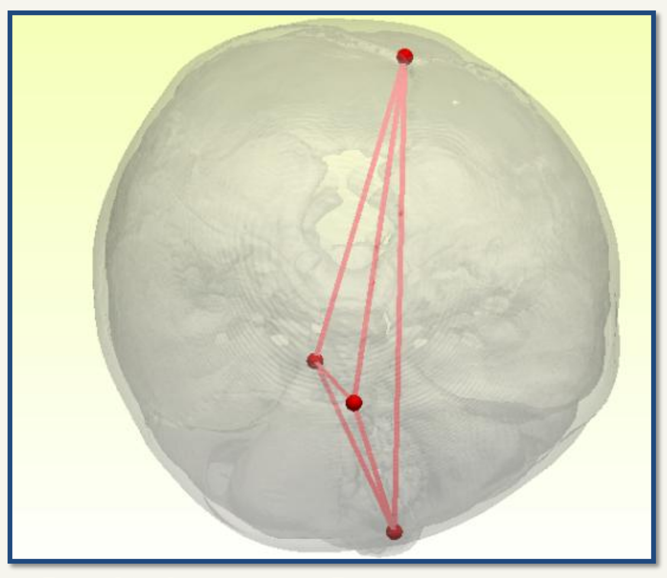
➢ K-means automatic classification into 4 clusters using the combinatorial distance matches the 4 diagnosis categories.

• Some Characterizations of Classes

(using only the 41 anatomical landmarks)

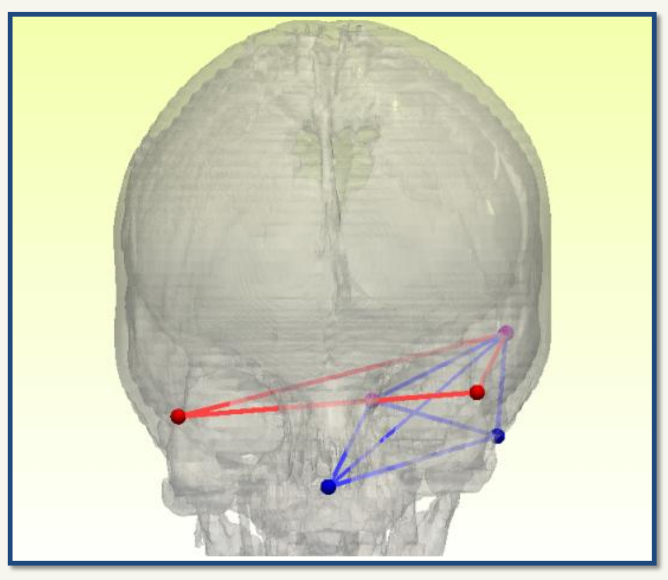


[$\chi(b_1) = +1$] ⇔ **RUCS**

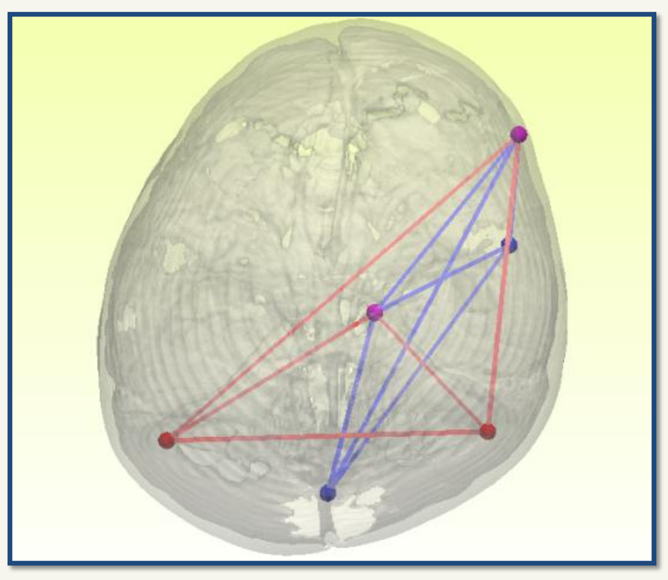


[$\chi(b_2) = +1$] ⇔ **LUCS**

➢ RUCS and LUCS are characterized by the sign of only 1 basis.
➢ The 2 basis b_1 and b_2 are symmetric w.r.t. the median sagittal plan.



[$\chi(b_3) = -1$]
and
[$\chi(b_4) = -1$]
⇔
BCS



➢ The signs of 2 bases characterize the category **BCS**.

➢ Based on the discriminability, we found a subset $\mathcal{S}$ of 5 bases and a vector x in $\{-1,1\}^{\mathcal{B}}$ such as: $\boldsymbol{M}$ is **unaffected** if and only if $\boldsymbol{M} \in B(\mathcal{S}, x, 2)$ (i.e. the signs of at least 3 of these 5 bases are the same in x and χ_M).